Calculating Trail Usage from Counter Data

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1. Introduction

Automatic counters are used on roads and trails to count the number of people or vehicles passing a fixed location. These counts are a measure of traffic and are of direct use to planners who are interested in traffic flow and congestion. For trails, however, it is usually the number of users, rather than traffic, which is of more importance.

The relationship between the number of counts and the number of users is not obvious. If there are several counters on a trail, some users may be counted several times, while others are completely missed. Although a great deal has been written on the subject of counting traffic (References [1], [2], [3]), almost nothing has been written on calculating the number of users.

In some cases, trail organizations have reported usage by simply summing the counts on the counters, with a possible correction for the fact that some users do an out-and-back trip. This procedure is clearly flawed, since by adding more counters one could increase the usage prediction to any level.

This report describes a new method for calculating trail usage from counter data. This new method shows that there is an elegantly simple relationship between counter values and number of users, namely

$$N_u = \frac{A}{\mu} \tag{1.1}$$

where N_u is the number of users, μ is the average distance traveled on the trail per use, and A is the area under the counter curve. An explanation and definition of what is meant by the “counter curve” is given in Section 2.

This new method has a number of significant features. Some of these are:

- It is simple to understand and use. The area under the counter curve is simply the total distance traveled on the trail by all trail users in the chosen time period.
- The calculations can be performed by hand or with a spreadsheet, rather than requiring a complex computer program.
- There are no assumptions or restrictions on how users travel the trail. They may do loops, retrace parts of their journey, or exit and reenter the trail at multiple points.
- It is no longer necessary to know the fraction of users who do a round trip.
- The method is applicable to a single trail or a network of trails.
- The method clarifies our understanding of how the accuracy of the usage prediction is influenced by the number and positions of the counters.

To understand the derivation of the method, it is important to fully understand the concept of a counter graph and the counter curve. These are described in the next section.

2. The Counter Graph

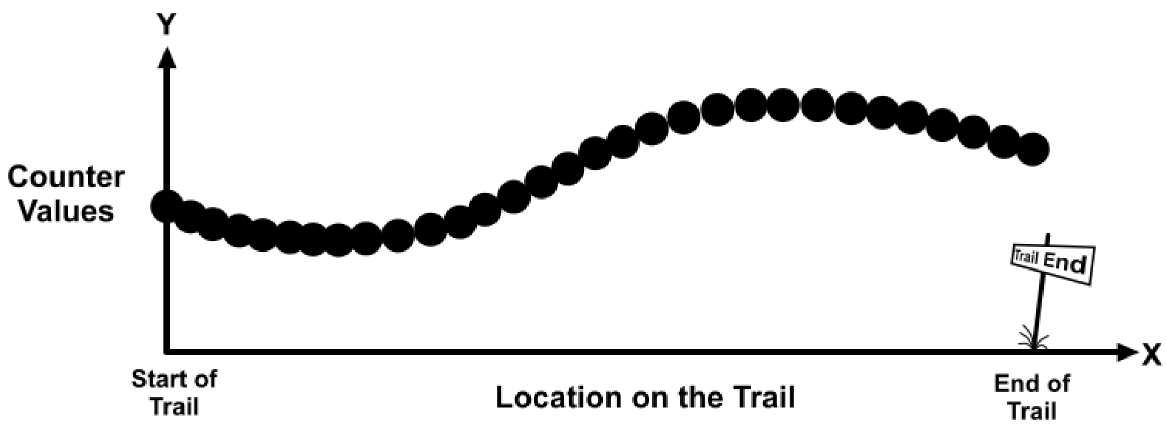
To understand this concept let us begin by assuming that we have a trail with three counters (numbered 1, 2, and 3) at points along the trail. Let the mile locations of the three counters be x_1 , x_2, and x_3. During a period of time (for example, 1 month) the counters will accumulate counts from users passing them. Let the counts on the three counters be C_1, C_2, and C_3, respectively.

We can now plot these three points on a graph as shown in Figure 1.

Figure 1 – A Graph of Three Counter Points

The vertical Y axis represents the counts recorded on the counters, and the horizontal X axis represents the position of each counter along the trail. We will refer to these points as counter points.

Next, suppose we are able to put many counters along the trail. The graph in this case might begin to look something like Figure 2.



As the number of counters increases we begin to see the formation of what appears to be a continuous curve. If we now take this concept to its limit and assume that there is a counter at every point on the trail, the graph will become a continuous curve[1], as shown in Figure 3. This curve will be called the counter curve.

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[1] Actually, since counts are integers the y values can only take on integer values. Therefore the curve is actually an integer-valued piecewise constant function with multiple jump discontinuities. However, after thousands of users it will begin to look smooth on a macroscopic level.

Figure 2 – A String of Counter Points

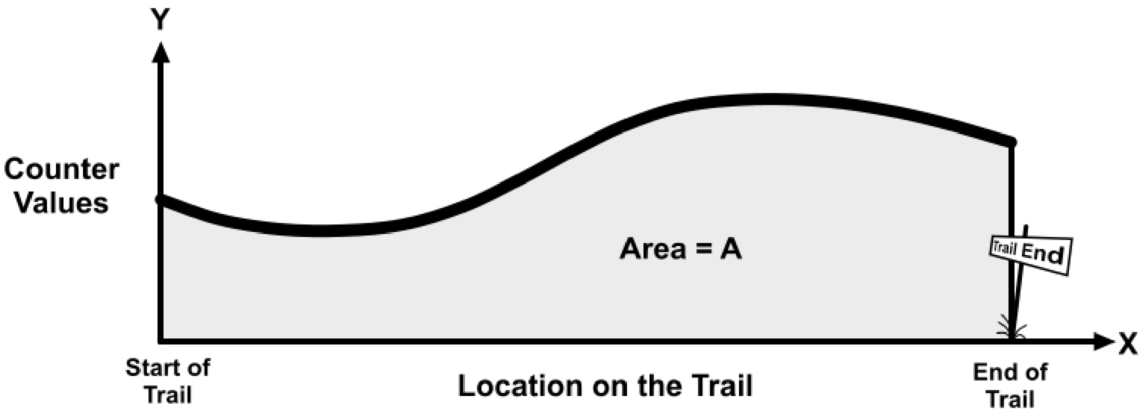

Figure 3 – The Counter Curve

The shape of the counter curve is determined by the readings on the counters during a specific time period. During another time period the curve will generally have different values and shape. The counter curve will be somewhat smooth because we would expect points on the trail near each other to have similar counts.

It should be noted that we can never fully know the shape of the counter curve because we will always have only a limited number of counters on the trail. These counters will only tell us a few points on the curve, not the whole curve.

It will be shown in the next section that the most important thing we need to know about the counter curve is the area under it. This area is the shaded region in Figure 3. It will be shown that the area under the counter curve is equal to the total distance traveled by all trail users. Therefore, the number of uses is just this total distance divided by the average distance traveled per use.

2. Derivation of the Area-Usage Equation

In this section we derive the relationship between the counter curve and the number of trail users. Assume that there is a counter at every point on the trail and assume that every counter initially reads zero. We will add one user at a time and see how the area under the counter curve is built up.

Consider our first user. Figure 4 shows his path from A to C to B to D.

Everywhere he goes he adds 1 to the counters as he passes them. On the graph, he is like a 1-count-wide paintbrush adding a gray stripe wherever he goes. If he doubles back onto his previous path it is as if he were taking a brush stroke just above his previous stroke.

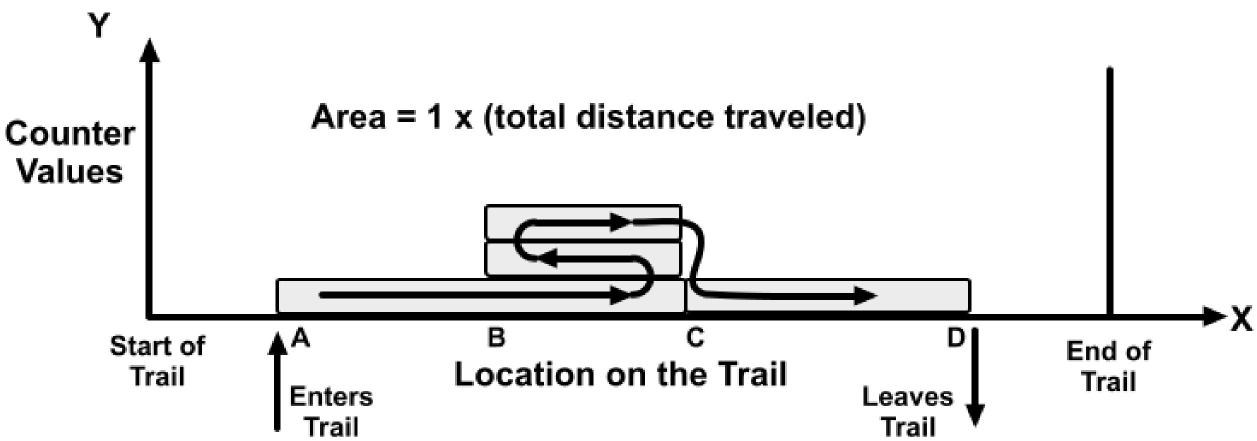

Figure 4 – The First User's Path

The shaded area is the area that he adds to the area under the graph. Since each rectangle is 1 unit tall, the total area that he adds under the curve is

$$A_1 = 1 \times \mathrm{d}_1 = \mathrm{d}_1 \tag{2.1}$$

where A_1 is the area added by user 1 and d_1 is the distance that he travels.

The next user will contribute a similar amount of area, given by

$$A_2 = 1 \times \mathrm{d}_2 = \mathrm{d}_2 \tag{2.2}$$

Continuing in this fashion, after N_u users have traveled the trail, the area, A, under the curve will be

$$A = (d_1 + d_2 + \cdots + d_{N_u}) \tag{2.3}$$

Equation (2.3) shows that the area under the counter curve is equal to the total distance traveled on the trail by all users in the given time period.

Dividing both sides of this equation by the number of users N_u gives

$$\frac{A}{N_u} = \frac{(d_1 + d_2 + \cdots + d_{N_u})}{N_u} \tag{2.4}$$

The term on the right side of the Equation (2.4) is just the sum of the distances traveled divided by the number of users. This is the average of the distances traveled. Therefore we can write this equation as

$$\frac{A}{N_u} = d_{ave} \tag{2.5}$$

where A is the total area under the counter curve, N_u is number of users, and d_{ave} is the average distance traveled by users (during this particular time period).

In estimating monthly usage, we are normally dealing with several thousand people. For samples of this size the average of the user distances during this period will be very close to the average, μ, of the distance traveled by the population of all trail users. Therefore we can substitute μ for d_{ave} in Equation (2.5) and rearrange the equation to give the desired Area-Usage Equation:

$$N_u = \frac{A}{\mu} \tag{2.6}$$

where N_u is the number of users, A is the area under the counter curve, and μ is the average distance traveled by all trail users. This is the fundamental equation connecting counter readings to trail usage.

It should be noted that the path taken by each user is completely arbitrary. They may go one way, retrace all or part of their path, or leave and return to the trail any number of times.

For illustrative purposes the diagrams above show a single linear trail. However, the equations are not restricted by this trail geometry. Equation (2.6) is equally valid for a network of trails. For a trail network the area in Equation (2.6) would be the sum of the graph areas for each trail segment in the network.

3. Approximating the Counter Curve Area

One of the main things we need to know to apply the Area-Usage Equation (2.6) is the area, A, under the counter curve. This is easy to calculate if we know the shape of the counter curve (i.e. its value at a large number of points). Like any physical quantity, we can determine the area under the counter curve to any desired degree of accuracy, provided we are willing to spend enough time and money to measure the counts at enough points on the trail. So the problem is an economic one, not a physics problem. Counters are expensive, so we want to do the best we can within our budget. This means trying to estimate the area under the counter curve with only a few counters on the trail.

Fortunately, mathematicians have been studying this subject for at least the last 300 years. In mathematical literature the estimation of the area under a curve from a knowledge of only a few points on the curve is called quadrature. It is a common problem in applied science and engineering and a number of methods for doing this estimation have been devised (see Reference [4]).

3.1 One-Counter Case: The Midpoint Method

We begin with the case where we only have one counter on the trail. The simplest approximation method is called the Midpoint Method. This method can be used when there is only one counter located preferably near the center of the trail. Figure 3 shows such a case where the one lone counter has a count of C. All we know is that this point lies on the unknown counter curve.

Since we don't have much to go on, we will initially just assume that the counter curve is a straight horizontal line through the counter point C. The area under this assumed counter curve is the area of the shaded rectangle shown. The area of this rectangular region is equal to the product of the length of the base times its height. The base is equal to the length of the trail L, and the height is the count value C.

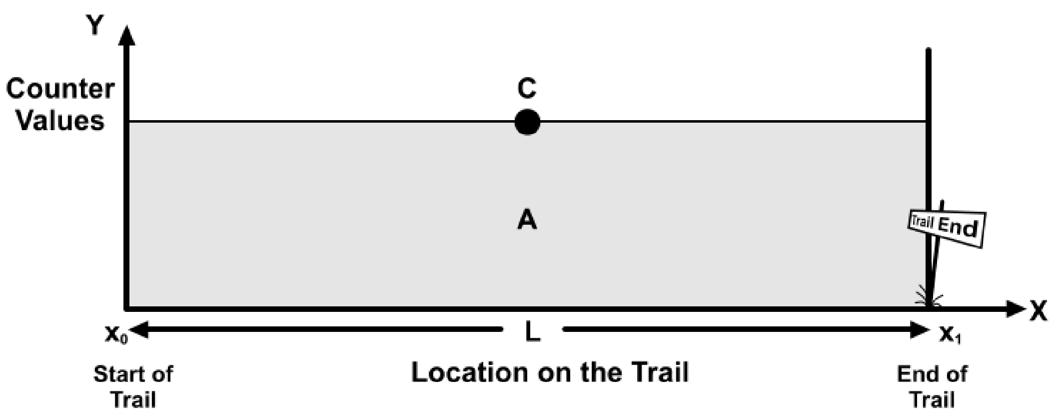

Figure 5

Therefore the area will be

$$A = LC \tag{3.1}$$

Choosing the counter curve to be a horizontal line probably seems like a very crude guess, and one that is not likely to be very close to the actual counter curve. But because we are choosing to measure the counts at the center of the trail, the formula for the area, Equation (3.1), does better than it would at first seem.

Suppose that instead of a horizontal line through the point C we choose a sloped line as shown in Figure 6.

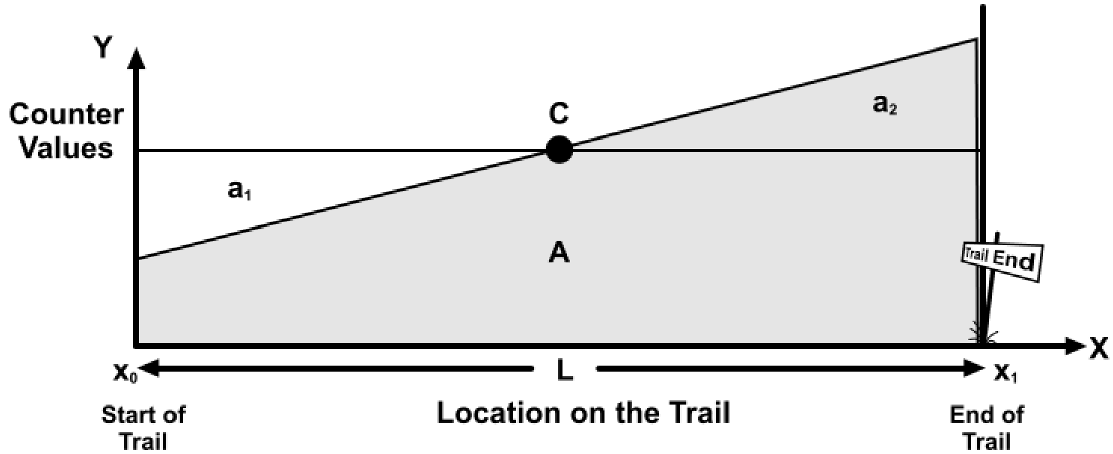

Figure 6 – A Sloped Curve Having the Same Area

The figure indicated by the shaded area is known as a trapezoid. Note that as the top tilts, the area under the sloped line (shaded area) stays the same as the rectangle because the area gained on the right (a_2) is the same as the area lost on the left (a_1).

What this means is that if the actual counter curve is approximately the same as any straight line passing through the mid-trail counter point, then the area under it will be approximately LC, where L is the length of the trail and C is the reading on the counter. We gained this extra capability to approximate the counter curve because we chose to place the counter in the center of the trail.

To get the number of trail users, recall that we just divide the area under the counter curve by the average distance traveled. With the area equal to LC, the formula for the usage can be written

$$N_u = \left(\frac{L}{\mu}\right) C \tag{3.2}$$

where N_u is the number of uses, L is the length of the trail, μ is the average distance traveled by trail users, and C is the count on the counter. Note that the term in parentheses in Equation (3.2) will not normally change from month to month and therefore one would only have to multiply the count on the counter by the same constant every month to calculate the number of users.

Example: Suppose a trail is 10 miles long, users average 5 miles per trip, and the one counter reads 3000 counts for the month. From equation (3.2) the usage for the trail will be approximately

$$N_u = \left(\frac{10}{5}\right)(3000) = 6000\ users \tag{3.3}$$

3.2 Two-Counter Case: Gauss' Method

The Midpoint Method described above showed that there are advantages to locating the counter at the center of the trail when there is one counter. The famous mathematician Carl Friedrich Gauss, who lived about 250 years ago, generalized this concept and developed methods that estimate the area when there are multiple counters. These methods, called Gaussian Quadrature, specify the optimum locations for the counters and provide a formula for calculating the area from the counter values. The downside of these methods is that we have to put the counters at very specific locations on the trail. If we can do this, then Gauss' methods can be a good choice for estimating the area under the counter curve.

Here, we will only look at the application of the method when there are two counters. The reader is referred to Reference [4] for the use of the method when there are more than two. We will not derive Gauss' method here, but simply explain how to use it.

As show in Figure 7, if the trail is of length L, Gauss' method requires that the counters be located at a distance of about $0.21 \times L$ from the ends of the trail.[2]

[2] More precisely, this distance is $\left(\frac{L}{2}\right)\left(1 - \frac{1}{\sqrt{3}}\right)$.

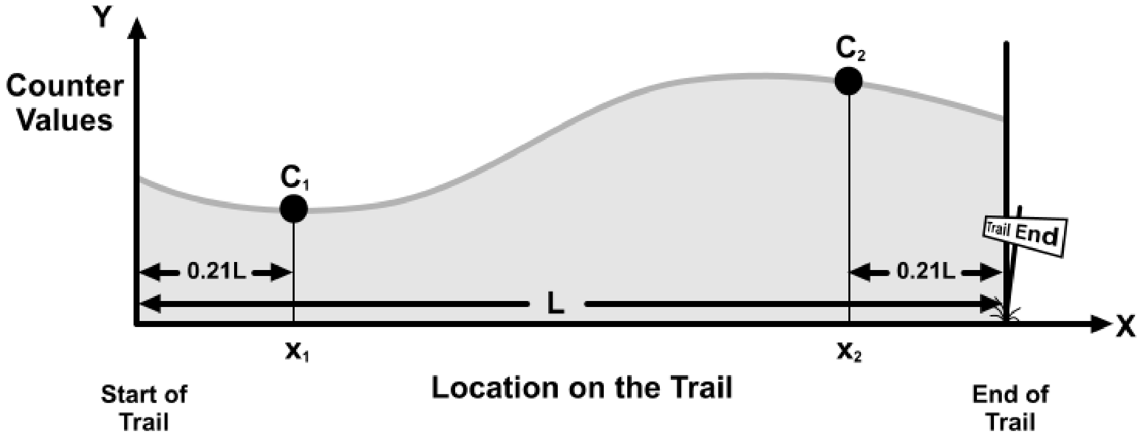

Figure 7 – Location of Counters with Gaussian Quadrature

If the counts on the two counters are C_1 and C_2, the area, A, under the curve will be approximately

$$A = L\left(\frac{C_1 + C_2}{2}\right) \tag{3.4}$$

where L is the length of the trail, and C_1 and C_2 are the counts on the counters.

This equation shows that the area under the counter curve is just the average of the counts on the two counters multiplied by the length of the trail.[3]

Recalling that the number of users is just this area divided by the average distance traveled, μ, the number of user, N_u, is given by

$$N_u = \left(\frac{L}{\mu}\right)\left(\frac{C_1 + C_2}{2}\right) \tag{3.5}$$

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[3] Note that we have not derived Equation (3.4) here, but simply stated the result. Derivations can be found in course notes available on the internet. Just search on "Two-Point Gaussian Quadrature." This simple formula provides a much more accurate estimate of the area than one would think. If the counter curve can be closely approximated by any cubic polynomial passing through the counter points, then Equation (3.4) will provide a good estimate of the area.

Example

Suppose the trail is 10 miles long, the average distance traveled by trail users is 5 miles, the count on counter #1 is 4000, and the count on counter #2 is 6000. Putting these numbers into Equation (3.5), the number of users, N_u is given by

$$N_u = \left(\frac{10}{5}\right)\left(\frac{4000 + 6000}{2}\right) = 10{,}000\ users \tag{3.6}$$

3.3 Multiple Counters at Arbitrary Locations

The Gaussian methods for estimating area under the counter curve provide good estimations; however, they require the counters to be at specific locations. This is not always practical. For example, infrared detection counters perform better when not in direct sunlight, and there is not always shade at the points Gauss would prefer.

So we next look at a method that does not require the counters to be at specific locations. The basic concept of this method is shown in Figure 8 where we have 5 counters along the trail, including one at each end.

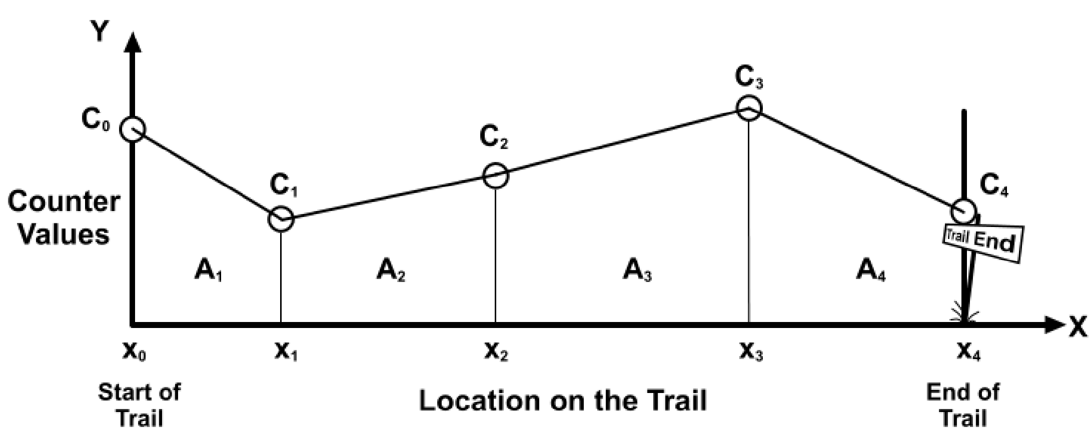

Figure 8 – Connecting the Counter Points

For this method we construct an approximation to the counter curve by simply connecting the counter points with straight lines. To calculate the area under this curve we observe that the area can be subdivided into four regions indicated by A_1, A_2, A_3, and A_4. Shapes of this type are called trapezoids, and this method is usually referred to as the

Trapezoidal Method. We can calculate the total area under the curve by calculating the area of each trapezoid and adding them together. The area of each of these trapezoids is just the width of their base multiplied by the average of the heights of the two sides. For example, the area A_1 is given by

$$A_1 = \left(\frac{C_0 + C_1}{2}\right)(x_1 - x_0) \qquad (3.7)$$

The above description is the normal form of the Trapezoidal Method. We're going to modify it a bit because for many trails people may not pass a counter at the ends because they turn around before reaching the end. So we will eliminate the end-of-trail counters and assume that if there were a counter at the ends, its count would be the same as the nearest counter. This type of counter curve approximation is shown in Figure 9.

As shown, the end regions become rectangles with heights C_1 and C_3.

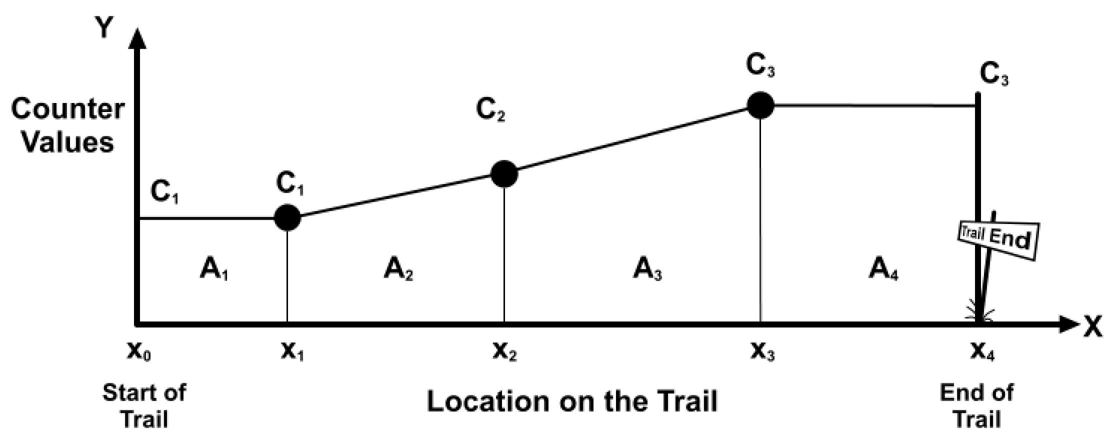

Figure 9 – Trapezoidal Method with No End Counters

For this case (and after a bit of algebra) it can be shown that the total area is given by

$$A = W_1 C_1 + W_2 C_2 + W_3 C_3 \tag{3.8}$$

where C_1, C_2, and C_3 are the counts on the counters and the weighting factors, W, are given by

$$W_1 = \left[-x_0 + \left(\frac{x_1 + x_2}{2}\right)\right] \tag{3.9}$$

$$W_2 = \left[\left(\frac{x_3 - x_1}{2}\right)\right] \tag{3.10}$$

$$W_3 = \left[x_4 - \left(\frac{x_3 + x_2}{2}\right)\right] \tag{3.11}$$

where the x coordinates are shown in Figure 9.[4]

Example:

Suppose the trail is 10 miles long, and the counters and trail ends are located at

$$x_0 = 0.0,\ x_1 = 2.5,\ x_2 = 5.0,\ x_3 = 7.5,\ \ x_4 = 10.0 \tag{3.12}$$

Assume the counts are:

$$C_1 = 2000,\ C_2 = 5000,\ C_3 = 3000 \tag{3.13}$$

From Equations (3.9), (3.10), and (3.11) we get

$$W_1 = 3.75,\ W_2 = 2.5,\ W_3 = 3.75 \tag{3.14}$$

[4] The corresponding equations for four or more counters are given in the Appendix.

and from Equation (3.8) the total area, A, is

$$A = (3.75)(2000) + (2.5)(5000) + (3.75)(3000) = 31{,}250 \tag{3.15}$$

If the average distance traveled is 5 miles, the total usage is obtained by dividing the area by the average distance traveled. The number of users, N_u, is then given by

$$N_u = \left(\frac{31{,}250}{5}\right) = 6250\; users \tag{3.16}$$

4. Summary

1. The method developed here is very easy to understand and use.
2. After defining the concept of the counter curve, it is easy to show that the area under the counter curve is equal to the distance traveled by all trail users in the chosen time period
3. This method calculates total trail usage only, whereas the previously-used computer models calculate both the usage and the distribution of usage along the trail. Normally it is only the total usage that is of primary interest.
4. Equation (2.5) is an exactly true equation. The approximation enters when we replace d_{ave}, the average user distance in a given time span, by μ, the average distance for all trail users, and approximate the area under the counter curve.
5. This method can be used to evaluate usage on networks of trails. A network could include loops, branches, and disconnected segments.
6. User behavior is completely arbitrary. All that is needed is the average distance traveled by all trail users. It is no longer necessary to estimate the percentage of people who do a round trip versus a one-way trip. The users may leave and re-enter the trail or reverse parts of their path any number of times on a single trip.

7. In a survey to determine the average distance traveled, μ, the important thing to ask is the actual distance on the trail that the user travels. The distance off the trail during their trip should not be included.
8. In this document, three methods for estimating the area under the counter curve have been demonstrated. There are a number of other methods that can be used. One of these other methods may be more appropriate or more accurate for the trail and counter configuration under consideration.

References

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2. National Cooperative Highway Research Program (NCHRP), (2014), Report 797,"Guidebook on Pedestrian and Bicycle Volume Data Collection," https://escholarship.org/content/qt11q5p33w/qt11q5p33w.pdf , (Accessed March 6, 2019)

3. National Cooperative Highway Research Program (NCHRP), (December 2016), Web only document ,"Methods and Technologies for Pedestrian and Bicycle Volume Data Collection: Phase 2," https://www.nap.edu/download/24732 " , (Accessed March 6, 2019)

4. Wikipedia contributors, "Numerical integration," *Wikipedia, The Free Encyclopedia,* https://en.wikipedia.org/w/index.php?title=Numerical_integration&oldid=883085876 (accessed February 15, 2019).

Appendix

The Modified Trapezoidal Method When There Are Four or More Counters

If a fourth counter is added in the future, the area equations become

$$A = W_1C_1 + W_2C_2 + \ W_3C_3 + W_4C_4 \tag{A.1}$$

$$W_1 = \left[-x_0 + \left(\frac{x_1 + x_2}{2}\right)\right] \tag{A.2}$$

$$W_2 = \left[\left(\frac{x_3 - x_1}{2}\right)\right] \tag{A.3}$$

$$W_3 = \left[\left(\frac{x_4 - x_2}{2}\right)\right] \tag{A.4}$$

$$W_4 = \left[x_5 - \left(\frac{x_4 + x_3}{2}\right)\right] \tag{A.5}$$

If you look at the above formulas for a while you can see the pattern that is developing. The first and last terms are different from the middle terms. The first W involves the first three coordinates, and the last W involves the last three coordinates. The middle W terms only have two coordinates, the first of which has an index 1 less than the W index, and the second has an index 1 greater than the W index. Therefore, we can write the equations for the general case for N counters as

$$A = W_1 C_1 + W_2 C_2 + \cdots + W_k C_k + \; W_{N-1} C_{N-1} + W_N C_N \tag{A.6}$$

where,

$$W_1 = \left[-x_0 + \left(\frac{x_1 + x_2}{2}\right)\right] \tag{A.7}$$

$$W_2 = \left[\left(\frac{x_3 - x_1}{2}\right)\right] \tag{A.8}$$

$$\cdots$$

$$W_k = \left[\left(\frac{x_{k+1} - x_{k-1}}{2}\right)\right] \tag{A.9}$$

$$\cdots$$

$$W_{N-1} = \left[\left(\frac{x_N - x_{N-2}}{2}\right)\right] \tag{A.10}$$

$$W_N = \left[x_{N+1} - \left(\frac{x_N + x_{N-1}}{2}\right)\right] \tag{A.11}$$